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Energy-Absorbing Characteristics of Foamed Polymers

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Synopsis

The energy-absorbing characteristics of a foam are determined by its load–compression response, and hence reflect the geometric structure and physical properties of the matrix material. In this report, the energy-absorbing characteristics are expressed in terms of three dimensionless quantities: (1) K, the energy-absorbing efficiency, (2) I, the impact energy per unit volume divided by E_f, and (3) I/K, the maximum decelerating force per unit area divided by E_f, where E_f is the apparent Young's modulus. Using the calculation procedures described in this report, it is now possible to delineate the geometric structure and physical properties a foam matrix must possess to meet a given energy absorption specification. This approach shows that: (1) the energy-absorbing characteristics of a brittle foam are superior to those of a ductile foam, (2) the optimum energy-absorbing foam has a large cell size, a narrow cell size distribution, and a minimum number of reinforcing membranes between the cells, (3) foam composites offer no significant advantage over a single foam, and (4) the optimum energy-absorbing region obtains over a tenfold change in impact velocity and can be extended in a given system only if the foam stiffness increases while the impact velocity is increased, as in a fluid-filled foam.

INTRODUCTION

An important commercial utilization of the compression behavior of foamed polymers is in the design of energy-absorbing structures. The ability to control load–compression response through variation of cell geometry, density, and matrix polymer makes a foamed polymer ideally suited for such applications. In many instances energy-absorbing materials are selected by empirical, trial-and-error procedures rather than by analytic techniques, because information relating the energy-absorbing characteristics to the critical foam variables is not available. To achieve the optimum design and material, these relationships must be understood quantitatively. The analytic techniques presented in this article make it possible to delineate precise relationships between foam parameters and energy absorption characteristics.

Using classical laws of motion, the impact energy absorption characteristics of a foam can be calculated from experimental compressive stress–strain data obtained at slow compression rates. Previous work[1,2] has demonstrated that the compressive stress, σ, can be factored into the product of two terms: (1) a dimensionless function of the compressive strain, $\psi(\epsilon)$, and (2) a factor ϵE_f, where E_f is the apparent Young's modulus of the foam and ϵ the compressive strain. E_f depends primarily on the volume fraction of polymer, φ, and on Young's modulus of the matrix polymer, E_0, and is largely independent of cell size or cell geometry. On the other hand, $\psi(\epsilon)$ reflects the buckling of the foam matrix and therefore is highly sensitive to the specific details of the matrix geometry, only moderately dependent on density or cell size, and independent of E_0 (and hence independent of temperature or strain rate). This procedure separates the influences of matrix geometry, density, and matrix polymer on the load–compression response and permits a convenient comparison between foams of widely different character.

The brittleness of the foam matrix also has a significant effect on $\psi(\epsilon)$. Because a brittle matrix is broken rather than flexed during compression, a brittle foam exhibits a load–compression curve with a flatter and wider plateau than that displayed by a ductile foam of equivalent E_f, φ, and matrix geometry. Therefore, it is important to distinguish between brittle foams (exhibiting glassy fracture) and "rigid" foams (exhibiting ductile fracture). Since both types appear equally stiff, this distinction, particularly important in energy-absorbing applications, frequently is not considered.

In the following sections of this article, the impact energy-absorbing characteristics of a foam are calculated from low-speed compressive σ–ϵ data written in terms of E_f and $\psi(\epsilon)$, assuming that $\psi(\epsilon)$ and E_f are independent of strain rate. These characteristics are expressed as relationships between three dimensionless quantities: (1) K, the energy-absorbing efficiency, (2) I, the impact energy per unit volume divided by E_f, and (3) I/K, the maximum decelerating force per unit area divided by E_f.

APPROACH

The most important characteristic of an energy-absorbing structure is the maximum deceleration, d_m, experienced by the impacting object, since an effective structure must absorb the total impact energy at a "safe" deceleration—a deceleration less than that which would cause damage to the impacting object. The maximum decelerating force is Md_m, where M is the mass of the impacting object.

It is generally agreed that an "ideal" energy-absorbing structure is one which deflects at a constant stress, hence providing a constant deceleration, for 100% of its thickness h. For such a structure, the kinetic energy of the impacting body, $Mv_i^2/2$, where v_i is the impact velocity, is equal to Md_mh; therefore

$$d_m = v_i^2/2h. \tag{1}$$

From this relationship we now can define the energy-absorbing efficiency,

K, as the inverse ratio of the maximum deceleration exhibited by a real material to that exhibited by an ideal material of equivalent thickness:

$$K = v_i^2/2hd_m. \tag{2}$$

K generally is expressed as a function of impact velocity. At low v_i, the impact energy is small relative to the stiffness of the foam, the degree of penetration is small, and K is low. At high v_i, the impact energy is large relative to the stiffness, the impacting body "bottoms" on the understructure, and K is low. At some intermediate v_i, K exhibits a maximum. The optimum material is one for which (1) the K-versus-v_i curve is as broad as possible, (2) $K_{\max}$ is close to unity, and (3) $K_{\max}$ occurs at the most probable v_i for the particular application.

In the present analysis K will be expressed as a function of I, a dimensionless quantity representing the impact energy per unit volume of foam divided by E_f. The maximum decelerating force per unit area of foam divided by E_f is I/K; this dimensionless quantity also will be expressed as a function of I.

The use of dimensionless quantities is emphasized throughout this report. This technique (1) minimizes the number of curves required to describe the energy-absorbing characteristics of a foam, (2) clearly shows the dependence of these curves on *all* of the critical parameters (such variables as the bulk dimensions of the foam frequently are ignored), and (3) provides a convenient method for comparing systems with widely different values for the critical parameters. Additionally, it permits a qualitative estimation of the energy-absorbing characteristics for a system which is difficult to calculate directly. For example, in this report calculations are presented for the uniform compression of a rectangular block of foam of area A and thickness h; few practical applications involve this simplified configuration. A more typical case would be a localized indentation in an irregularly shaped piece of foam. Although A and h cannot be specified for this latter case, it still can be assumed that the K-versus-I and the I/K-versus-I curves depend on $\psi(\epsilon)$ and E_f qualitatively in the same manner as for the simplified configuration.

ANALYTIC SCHEME

For a foamed material, the compressive stress σ can be expressed[1,2] as follows:

$$\sigma = \epsilon E_f \psi(\epsilon) \tag{3}$$

where $\psi(\epsilon)$ is a dimensionless function of the compressive strain ϵ and E_f is the apparent Young's modulus. Considering the uniform compression of a block of foam with a cross-sectional area A and thickness h, the maximum deceleration is $A\sigma_{\max}$ (generally the stress at the maximum degree of compression) divided by M, the mass of the impacting body. Therefore, from eq. (3) we obtain

$$d_m = AE_f\epsilon^*\psi(\epsilon^*)/M \tag{4}$$

where ϵ^* is the ultimate compressive strain. The energy absorbed by the foam is the area under the load–compression curve and is equal to $Mv_i^2/2$, the kinetic energy of the impacting body:

$$Mv_i^2/2 = AhE_f \int_0^{\epsilon^*} \psi(\epsilon)\ \epsilon\ d\epsilon. \tag{5}$$

Thus, from the definition of K and eqs. (4) and (5) is obtained

$$K = \int_0^{\epsilon^*} \psi(\epsilon)\ \epsilon\ d\epsilon / \epsilon^* \psi(\epsilon^*). \tag{6}$$

During impact, compression rate decreases as foam penetration increases. For simplicity, eq. (5) assumes that the load–compression behavior is independent of impact velocity, that is, $\psi(\epsilon)$ and E_f are not a function of strain rate. Since $\psi(\epsilon)$ reflects primarily matrix buckling, it would not be expected to exhibit a strong rate dependence unless the increased strain rate caused the matrix polymer to undergo a ductile–brittle transition.[1,2] The matrix modulus E_0 will display a characteristic rate dependence, but this strain rate dependence is negligible for most foams. Pneumatic damping represents the most important rate-dependent process occurring during compression of a foam.[3] This mechanism is significant, however, only if E_f is less than about 100 psi and the foam permeability is very low.[4] Therefore, the assumptions that impact velocity has no significant influence on the compressive stress–strain curve and that impact energy-absorbing characteristics can be calculated from low-speed compression data are justified for most foamed polymers (particularly rigid, brittle foams).

Equation (5) suggests that K should be expressed as a function of the dimensionless quantity $(Mv_i^2/2Ah)/E_f$. This quantity will be denoted I and represents the impact energy per unit volume of foam divided by E_f:

$$(Mv_i^2/2Ah)/E_f = I = \int_0^{\epsilon^*} \psi(\epsilon)\ \epsilon\ d\epsilon. \tag{7}$$

From eq. (4) it can be seen that the maximum decelerating force per unit area of foam divided by E_f is the dimensionless quantity I/K:

$$(Md_m/A)/E_f = I/K = \epsilon^* \psi(\epsilon^*). \tag{8}$$

For a given block of foam and a given impacting mass (M, E_f, A, and h constant), a plot of K against I represents the dependence of the energy-absorbing efficiency on the impact velocity. Similarly, the I/K-versus-I curve represents the dependence of the maximum deceleration on impact velocity. The detailed shapes of these characteristic curves are dependent only on the function $\psi(\epsilon)$, while the location of these curves along the impact energy and maximum deceleration axes is dependent on E_f and the bulk dimensions of the foam. Previous work has established the dependence of $\psi(\epsilon)$ and E_f on the critical foam parameters.[1,2] Therefore, we can conclude that the detailed shapes of the characteristic energy absorption curves are dependent primarily on the geometric structure and

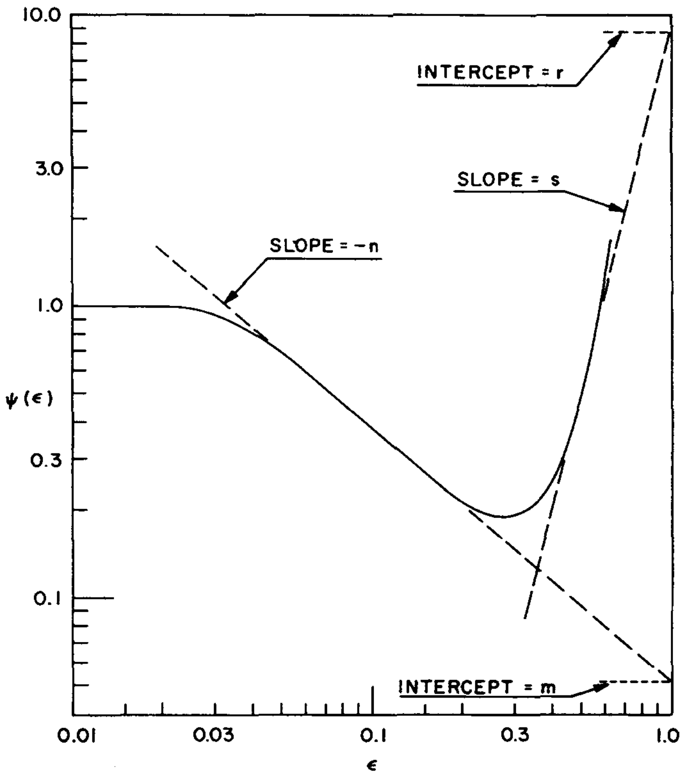

Fig. 1. Evaluation of constants m, n, r, and s from typical experimental data for $\psi(\epsilon)$.

brittleness of the polymer matrix and that the location of these curves along the I and I/K axes depends primarily on the volume fraction of polymer, φ, and the modulus of the base polymer, E_0.

In the absence of a ductile–brittle transition, $\psi(\epsilon)$ is essentially independent of temperature or compression rate.[1,2] Thus, the influence of temperature on the energy absorption behavior arises solely from changes in E_f and the temperature dependence can be expressed as a shift factor along the I and I/K axes.

Once $\psi(\epsilon)$ has been determined experimentally, the characteristic K-versus-I and I/K-versus-I curves can be obtained by selecting appropriate values of ϵ^* and calculating K and I from eqs. (6) and (7). Unfortunately, $\psi(\epsilon)$ is not a simple function and the exact calculation cannot be done analytically. For many purposes, however, the following analytic expression for $\psi(\epsilon)$ is sufficiently accurate:

$$\psi(\epsilon) = m\epsilon^{-n} + r\epsilon^{s} \tag{9}$$

where m, n, r, and s are curve-fitting constants. The evaluation of these constants from a typical logarithmic plot of $\psi(\epsilon)$ against ϵ is illustrated in Figure 1. From eq. (9), it now follows that

$$I = [m/(2 - n)]\epsilon^{*2-n} + [r/(2 + s)]\epsilon^{*2+s} \quad (10)$$

and

$$K = I/[m\epsilon^{*1-n} + r\epsilon^{*1+s}] \quad (11)$$

Equation (9) is applicable only at $\epsilon > \epsilon_b$. At compressions less than ϵ_b, $\psi(\epsilon) \backsimeq 1.0$. Thus

$$I = \epsilon^{*2}/2 \quad (12)$$

and

$$K = \epsilon^{*}/2. \quad (13)$$

Composite Foam Systems

An energy-absorbing structure may be a composite of two or more different foams. In this section, K and I are calculated for the uniform compression of a rectangular composite structure, considering both a "series" (strains additive) and a "parallel" (stresses additive) configuration.

For a series configuration, comprised of foams "1" and "2" of equal cross-sectional area A and combined thickness h, the compressive stress on each foam is identical and the total compressive strain is the sum of ϵ_1 and ϵ_2. The following equations, analogous to eqs. (4) and (5), can be written

$$d_m = (A/M)[\epsilon_1^*\psi_1(\epsilon_1^*)E_{f_1}] = (A/M)[\epsilon_2^*\psi_2(\epsilon_2^*)E_{f_2}] \quad (14)$$

and

$$Mv_i^2/2 = Ah_1E_{f_1}\int_0^{\epsilon_1^*}\psi_1(\epsilon_1)\,\epsilon_1\,d\epsilon_1 + Ah_2E_{f_2}\int_0^{\epsilon_2^*}\psi_2(\epsilon_2)\,\epsilon_2\,d\epsilon_2 \quad (15)$$

From the definitions for I and K given previously, we now obtain

$$I^* = (Mv_i^2/2Ah)/E_{f_2} = [\alpha\beta/(\alpha + 1)]\int_0^{\epsilon_1^*}\psi_1(\epsilon_1)\,\epsilon_1\,d\epsilon_1 + [1/(\alpha + 1)]\int_0^{\epsilon_2^*}\psi_2(\epsilon_2)\,\epsilon_2\,d\epsilon_2 \quad (16)$$

and

$$K^* = I^*/[\epsilon_2^*\,\psi_2(\epsilon_2^*)] \quad (17)$$

where $\alpha = h_1/h_2$, $\beta = E_{f_1}/E_{f_2}$, $h = h_1 + h_2$, and I^* and K^* represent the values characteristic of the composite. From eq. (14), ϵ_1^* and ϵ_2^* are related by the expression

$$\beta\,\epsilon_1^*\,\psi_1(\epsilon_1^*) = \epsilon_2^*\,\psi_2(\epsilon_2^*). \quad (18)$$

From experimental data for $\psi_1(\epsilon)$ and $\psi_2(\epsilon)$ and selected values for α and β, the K^*-versus-I^* curve (or the I^*/K^*-versus-I^* curve) is obtained by choosing appropriate values for ϵ_1^*, calculating ϵ_2^* from eq. (18), and then calculating I^* and K^* from eqs. (16) and (17).

In the case of a parallel configuration, comprised of foams "1" and "2" of equal thickness h and combined cross-sectional area A, the compressive strain is identical for each foam and the total compressive force is the sum of $\sigma_1 A_1$ and $\sigma_2 A_2$. The following equations can be written for this system:

$$d_m = (A_1/M)\ [\epsilon^*\psi_1(\epsilon^*)E_{f_1}] + (A_2/M)\ [\epsilon^*\psi_2(\epsilon^*)E_{f_2}] \tag{19}$$

and

$$Mv_i^2/2 = A_1hE_{f_1}\int_0^{\epsilon^*} \psi_1\ (\epsilon)\ \epsilon\ d\epsilon + A_2hE_{f_2}\int_0^{\epsilon^*} \psi_2(\epsilon)\ \epsilon\ d\epsilon. \tag{20}$$

From the definitions for I and K, we now obtain

$$I^* = (Mv_i^2/2Ah)/E_{f_2}$$

$$= [\gamma\beta/(\gamma + 1)]\int_0^{\epsilon^*} \psi_1(\epsilon)\ \epsilon\ d\epsilon + [1/(\gamma + 1)]\int_0^{\epsilon^*} \psi_2(\epsilon)\ \epsilon\ d\epsilon \tag{21}$$

and

$$K^* = (\gamma + 1)I^*/[\gamma\beta\epsilon^*\psi_1(\epsilon^*) + \epsilon^*\psi_2(\epsilon^*)] \tag{22}$$

where $\gamma = A_1/A_2$ and $A = A_1 + A_2$.

From experimental data for $\psi_1(\epsilon)$ and $\psi_2(\epsilon)$ and selected values for β and γ, the K^*-versus-I^* and I^*/K^*-versus-I^* curves are obtained by choosing appropriate values for ϵ^* and calculating I^* and K^* from eqs. (21) and (22).

RESULTS AND DISCUSSION

From previously reported experimental data for $\psi(\epsilon)$ and E_f[1,2] the dimensionless quantities K, I, and I/K were calculated using eqs. (10)–(13). The characteristic K-versus-I and I/K-versus-I relationships for several typical foams (Table I) are shown in Figures 2–4. The terminal point on these curves, represented by a circle, is the point at which the

TABLE I
Constants for the Calculation of $\psi(\epsilon)$

Sample	Type	φ	E_f, psi	m	n	r	s
C (25°C)	polyurethane	0.11	25	0.20	0.58	6.0	6.5
C (−196°C)	polyurethane	0.11	1,500	0.11	1.0	1.2	4.7
G (25°C)	polyurethane	0.033	15	0.070	0.89	2.0	13.
G (−196°C)	polyurethane	0.033	750	0.057	1.0	0.70	11.
S	polyurethane	0.037	740	0.045	0.95	6.0	22.
V	polyurethane	0.30	17,500	0.060	0.89	2.2	6.7
Z	polystyrene	0.020	370	0.070	0.75	1.0	7.5
AA	polyethylene	0.033	110	0.15	0.65	1.8	2.9
DD	ABS	0.44	42,000	0.050	0.89	8.0	3.8
GG	phenolic	0.032	1,050	0.023	1.0	1.0	16.

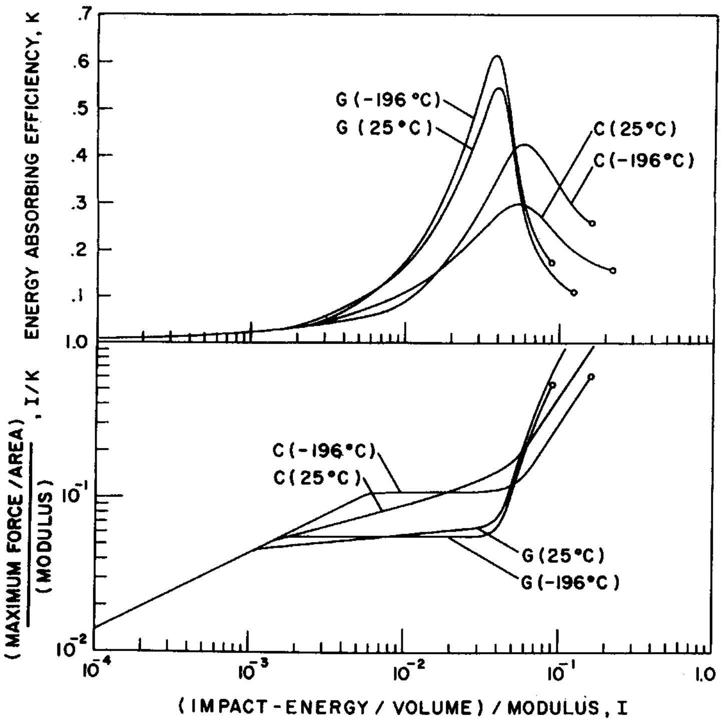

Fig. 2. Dependence of characteristic energy absorption curves on matrix brittleness for two low-density polyurethane foams.

foam is completely compressed (the impacting object "bottoms" on the understructure); if the understructure is infinitely rigid, the maximum deceleration rises rapidly to infinity and the efficiency drops to zero. The constants m, n, r, and s required to approximate $\psi(\epsilon)$ by eq. (9) are given in Table I.

Figure 2 illustrates the influence of matrix brittleness on the characteristic energy absorption curves for two typical polyurethane foams. These curves are calculated from compression data at 25°C (flexible matrix) and −196°C (brittle matrix). For an identical matrix geometry, the brittle foam exhibits a higher efficiency (in the region of maximum energy absorption) and a flatter, wider plateau in the deceleration-versus-impact energy curve than the flexible foam. The large change in E_f produced by this temperature difference serves only to shift the position of these curves along the impact energy and maximum deceleration axes; the detailed shapes of these curves depends only on $\psi(\epsilon)$.

Additional examples of the effect of matrix brittleness are given in Figures 3 and 4. Samples S (polyurethane) and GG (phenolic) with a very brittle matrix, sample AA (polyethylene) a very ductile matrix, and sample Z (polystyrene) a semiductile matrix, are typical examples of low-

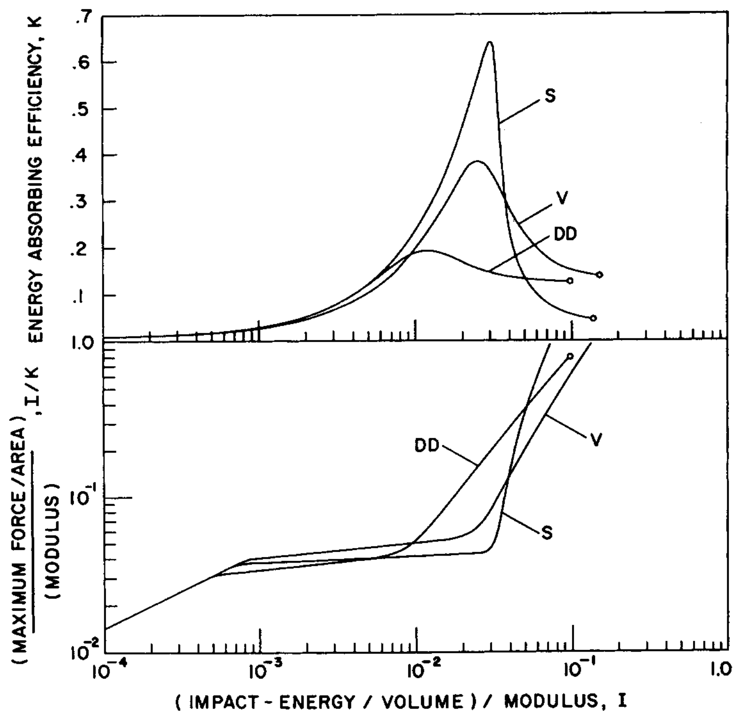

Fig. 3. Dependence of characteristic energy absorption curves on properties of matrix polymer. See Table I for sample symbols.

density foams (φ = 0.02–0.04). A comparison for high-density foams (φ = 0.3–0.5) is provided by sample V (polyurethane) with a brittle matrix and sample D (ABS) with a ductile matrix. The calculated curves clearly show the superior energy-absorbing properties exhibited by brittle, as opposed to ductile or flexible, foams. The difference is more pronounced for high-density foams. This result is particularly important for applications where the impact energies are large.

The detailed shapes of the characteristic energy-absorption curves are governed by $\psi(\epsilon)$ and hence are dependent on matrix geometry as well as brittleness. To obtain a high K and a flat, wide plateau in the I/K-versus-I curve, (1) $\psi(\epsilon)_{\min}$ should be low, (2) the difference between ϵ at $\psi(\epsilon)_{\min}$ and ϵ_b should be large, and (3) the logarithmic slope of $\psi(\epsilon)$ against ϵ should be -1.0 (n = 1.0). Therefore, the optimum energy-absorbing foam has a brittle matrix with a large cell size, a narrow cell-size distribution, and a minimum number of reinforcing membranes between cells.

Composite Foam Systems

The characteristic K^*-versus-I^* and K^*/I^*-versus-I^* curves were calculated for several representative series and parallel foam composites; these

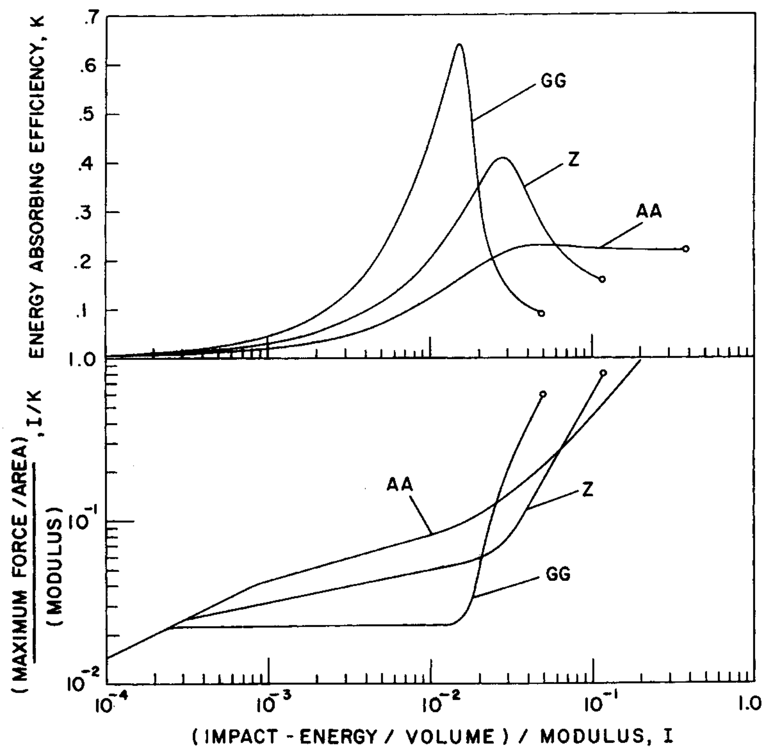

Fig. 4. Dependence of characteristic energy absorption curves on properties of matrix polymer. See Table I for sample symbols.

are presented in Figures 5–8. The required $\psi(\epsilon)$ functions were calculated from the m, n, r, and s constants in Table I, but the parameter β was selected arbitrarily. The constants α and γ were taken as unity in all cases, since the qualitative influence of these parameters on the calculated curves is obvious. For comparison, the characteristics of each foam component individually are indicated by broken curves. The point at which the composite is completely compressed is indicated by a circle.

Figures 5 and 6 represent the case of two foams with similar $\psi(\epsilon)$ functions but different modulii. The series composite exhibits two separate peaks in the efficiency curve and two separate plateaus in the maximum deceleration curve. Thus, each foam tends to behave independently, but at a reduced efficiency. This situation is not desirable for most applications. The parallel composite exhibits a single efficiency peak and a single maximum deceleration plateau, which are similar to those characteristic of each component individually, but shifted along the impact energy and deceleration axes. Thus, this type of composite would be useful for shifting the position of the energy absorption curves (if changing E_0 or φ is impractical), although the shape is not altered.

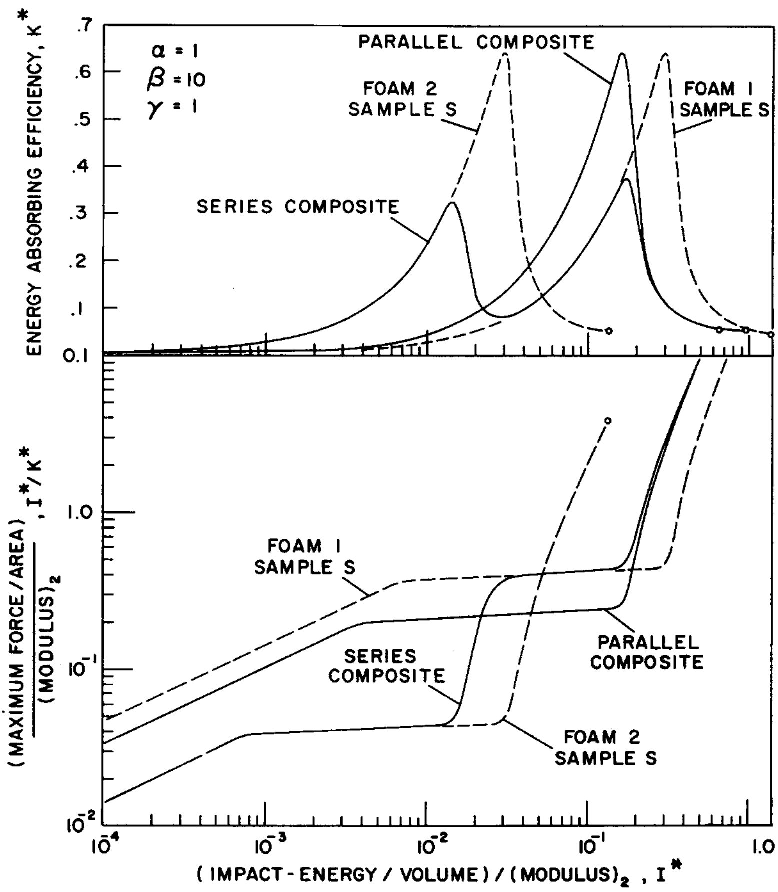

Fig. 5. Characteristic energy absorption curves for hypothetical foam composite.

Figures 7 and 8 represent the case of two foams with different $\psi(\epsilon)$ functions. As in the previous examples, each foam in the series composite reacts independently at a reduced efficiency, while the response of the parallel composite is an average of the two foam component. For either composite configuration, although the shapes of the characteristic curves are changed, the efficiency is not increased, and the maximum deceleration plateau is neither wider or flatter than that of one of the components. Therefore foam composites cannot be expected to offer an improvement in energy-absorbing properties over those of the best component in the composite. A parallel composite may be used to shift the response, but generally it would be more practical to change E_0 or foam density.

Optimum Energy Absorption

The width of the optimum energy-absorbing region can be no greater than 2 decades along the I axis for any realistic variation in $\psi(\epsilon)$. For a

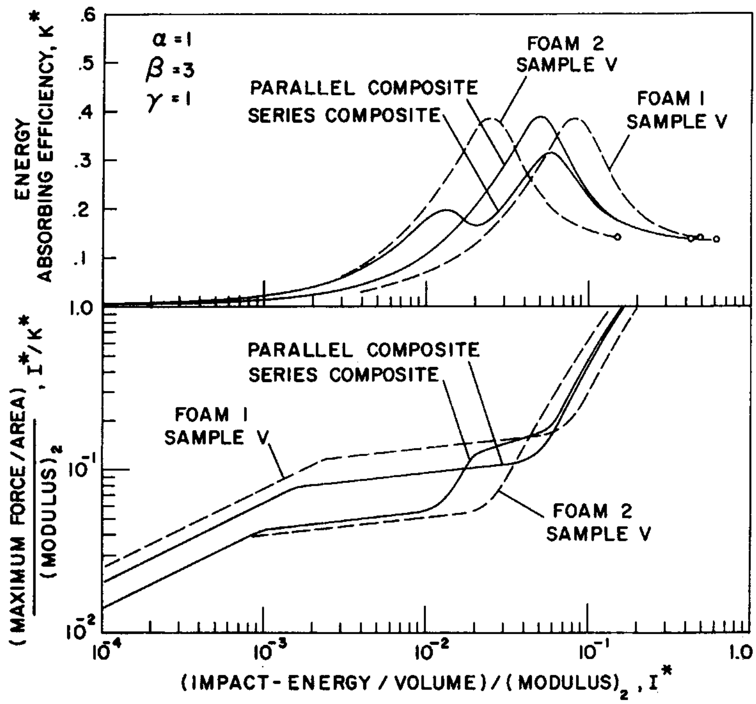

Fig. 6. Characteristic energy absorption curves for hypothetical foam composite.

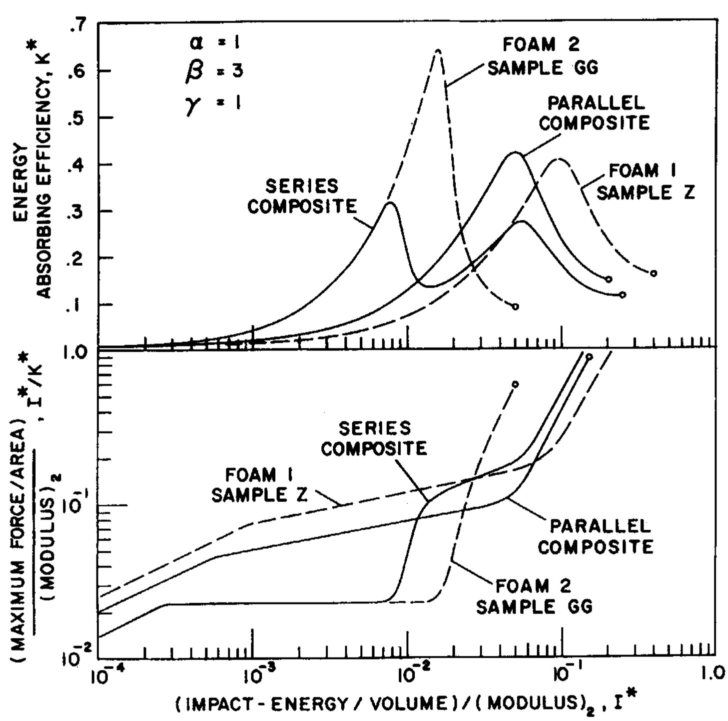

Fig. 7. Characteristic energy absorption curves for hypothetical foam composite.

typical case where E_f, M, and bulk dimensions are constant, this is only a factor of 10 in impact velocity. For many applications this is not large enough.

The only practical way the optimum absorption region can be expanded, in terms of impact velocity dependence, is to make the apparent compressive stiffness of the foam, E_f, increase as impact velocity increases. While polymers do exhibit an increasing modulus with increasing strain rate, this rate dependence is large over a limited temperature range only, and in this range the modulus also is strongly temperature dependent. Most applications would require that E_f be essentially independent of temperature, and therefore the viscoelastic properties of the matrix polymer cannot be employed to expand the optimum absorption region. A mechanism has been described[3] by which the apparent compressive stiffness of a foam can be made highly strain-rate dependent while remaining nearly independent of temperature. This is accomplished by controlling the rate of fluid flow (either air or liquid) through the foam matrix during compression. In this type of system, the apparent stiffness at impact is approximately proportional to the impact velocity squared, where the proportionality constant is dependent on fluid density, foam density, matrix geometry, and bulk dimensions of the foam. From eq. (7) it now can be seen that for a given mass and foam geometry, I is a constant independent of the impact

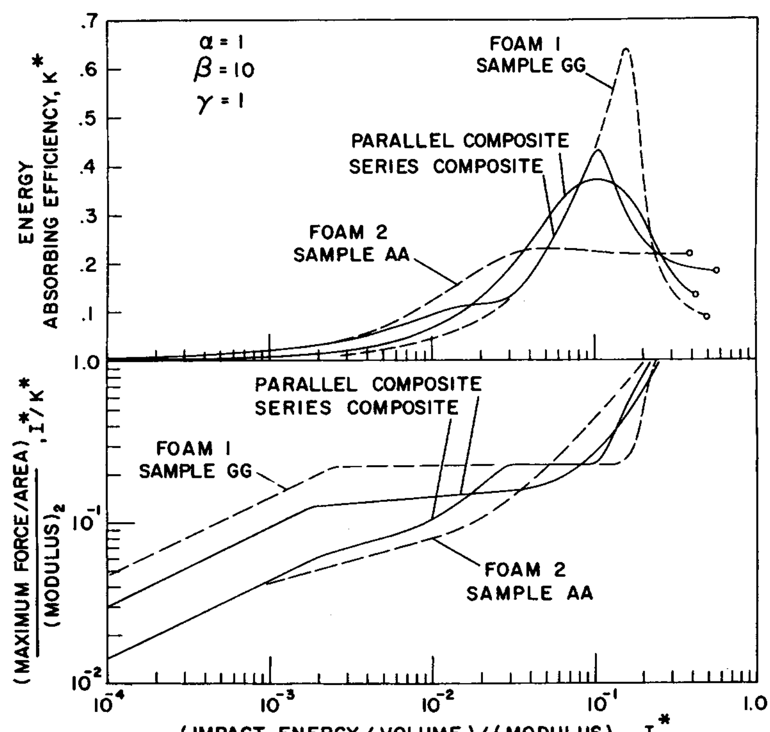

Fig. 8. Characteristic energy absorption curves for hypothetical foam composite.

velocity. In principle, therefore, a fluid-filled foam system could be designed which would possess an approximately constant efficiency and maximum deceleration over a very wide impact velocity range.

CONCLUSIONS

(1) The energy-absorbing characteristics of most foams can be calculated from experimental data for $\psi(\epsilon)$ and E_f obtained from low-speed compression tests. These characteristics are expressed in terms of three dimensionless quantities: (1) K, the energy-absorbing efficiency, (2) I, the impact energy per unit volume divided by E_f, and (3) I/K, the maximum decelerating force per unit area divided by E_f. The detailed shapes of the K-versus-I and I/K-versus-I curves are dependent only on $\psi(\epsilon)$, while E_f and the bulk dimensions serve as shift factors along the impact energy and maximum deceleration axes.

(2) In the region of maximum energy absorption, a brittle foam exhibits a higher energy-absorbing efficiency and a wider, flatter plateau in the maximum deceleration-versus-impact energy curve than a ductile (or flexible) foam of similar matrix geometry. This difference is more pronounced for high density than low density foams. Additionally, the optimum energy-absorbing foam has a large cell size, a narrow cell-size distribution, and a minimum number of reinforcing membranes between cells.

(3) No significant advantage, in terms of the energy-absorbing properties, is gained by preparing a composite foam system. In a series composite, each foam component behaves independently at a reduced efficiency, and in a parallel composite the properties are an average of those characteristic of the individual components.

(4) To produce an energy-absorbing system with a high efficiency over a broad range of impact velocities, the apparent stiffness of the foam must increase with impact velocity, but remain independent of temperature. A fluid-filled foam in which the principal energy dissipation process is the flow of fluid through the open-cell matrix does display this type of behavior.

It is a pleasure to acknowledge Dr. S. Newman for many helpful discussions during the course of this work, and Drs. A. J. Chompff and S. Newman for their careful review of this manuscript.

List of Symbols

A	Cross-sectional area perpendicular to compression-direction.
d_m	Maximum deceleration.
E_f	Apparent Young's modulus of the foam.
E_0	Young's modulus of the matrix polymer.
h	Thickness of foam parallel to compression direction.
I	Dimensionless quantity representing the impact energy per unit volume of foam divided by E_f.

I^*	Value of I for a foam composite.
I/K	Dimensionless quantity representing the maximum decelerating force per unit area of foam divided by E_f.
I^*/K^*	Value of I/K for a foam composite.
K	Dimensionless quantity representing the energy-absorbing efficiency.
K^*	Value of K for a foam composite.
K_{max}	Maximum value of K.
M	Mass of the impacting object.
m	Curve-fitting constant in eq. (9).
n	Curve-fitting constant in eq. (9).
r	Curve-fitting constant in eq. (9).
s	Curve-fitting constant in eq. (9).
v_i	Impact velocity.
α	Ratio h_1/h_2 for a foam composite.
β	Ratio E_{f_1}/E_{f_2} for a foam composite.
γ	Ratio A_1/A_2 for a foam composite.
ϵ	Compressive strain.
ϵ^*	Ultimate compressive strain during impact.
ϵ_b	Critical buckling strain [$\psi(\epsilon) = 0.95$].
φ	Volume fraction of polymer.
σ	Compressive stress.
$\psi(\epsilon)$	Dimensionless function of ϵ calculated from experimental load–compression data.
$\psi(\epsilon)_{min}$	Minimum value of $\psi(\epsilon)$.
$_1$	Subscript used to denote foam "1" in a composite.
$_2$	Subscript used to denote foam "2" in a composite.

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